TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

logics Logical Methods in Computer Science

Microsoft

Interpolants from SAT solving certificates

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Martin Matak
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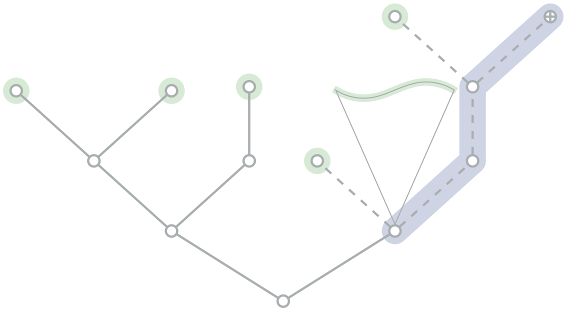
TU Wien



Helmut Veith Workshop
Obertauern, Austria

Janary 31, 2018

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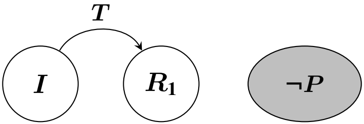
Interpolation-based Image Approximation



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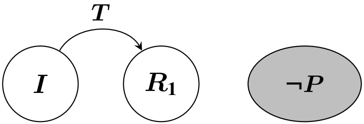

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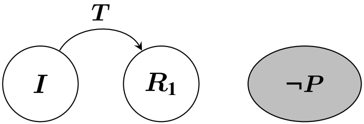

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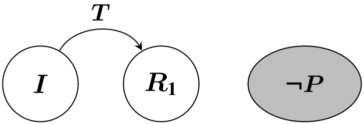

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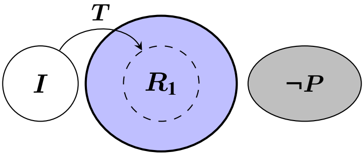

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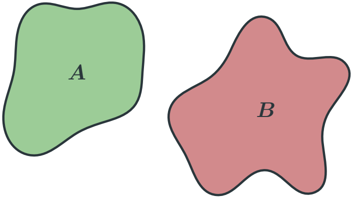
Propositional interpolants

Let A, B be propositional formulae such that $A \wedge B$ is unsatisfiable



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- $A \vDash P$
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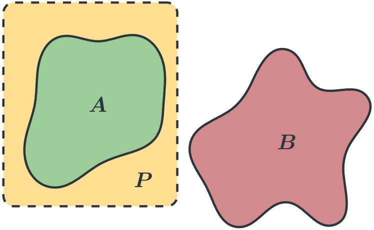

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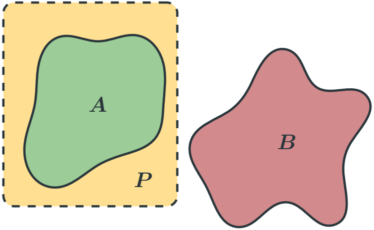

Interpolants are essential tools in formal methods and software verification:

- (Un)bounded model checking [McMillan '03]
- Boolean synthesis [Jiang et al. '09]
- Fault localization [Ermis et al. '12]
- Hardware verification [Keng Veneris '09]

Interpolation in practice

Interpolant generation

The good old times...

Interpolant generation

The good old times are gone

Interpolant generation

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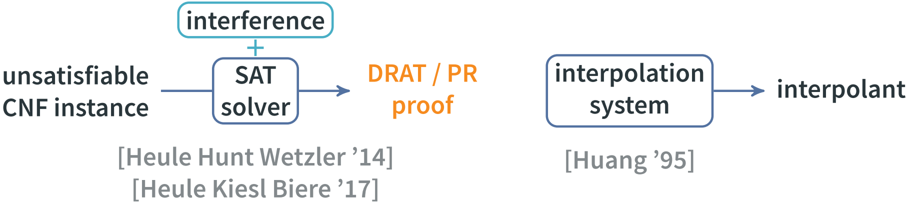


Properties of DRAT / PR proofs

- ✔ Shorter and easier to generate or check than resolution proofs
- ✔ Allow to express satisfiability-preserving techniques

Interpolant generation

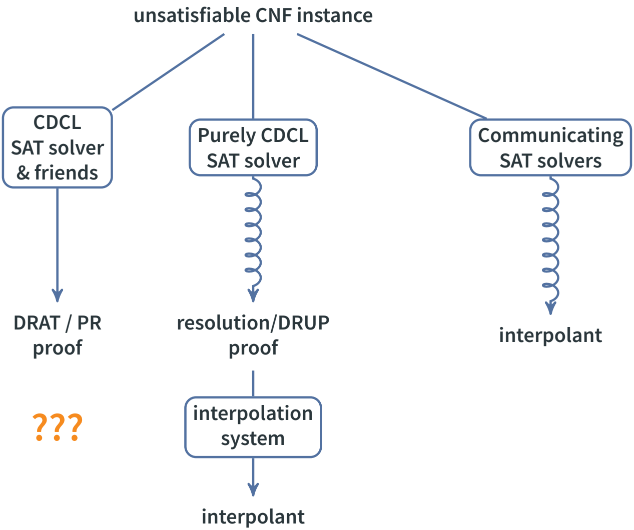
The good old times are gone

Properties of DRAT / PR proofs

- ✔ Shorter and easier to generate or check than resolution proofs
- ✔ Allow to express satisfiability-preserving techniques
- ✘ We do not know how to generate interpolants from DRAT / PR proofs

Interpolant generation from proofs

Three approaches



Interpolant generation from proofs

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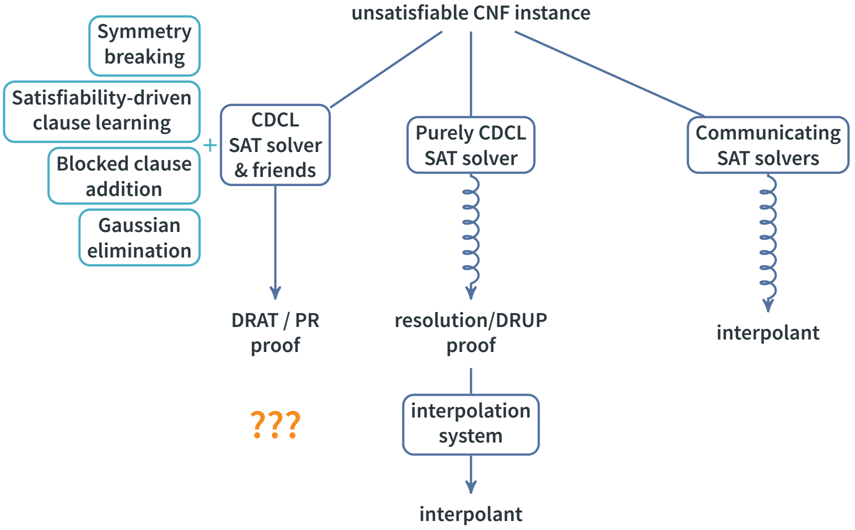

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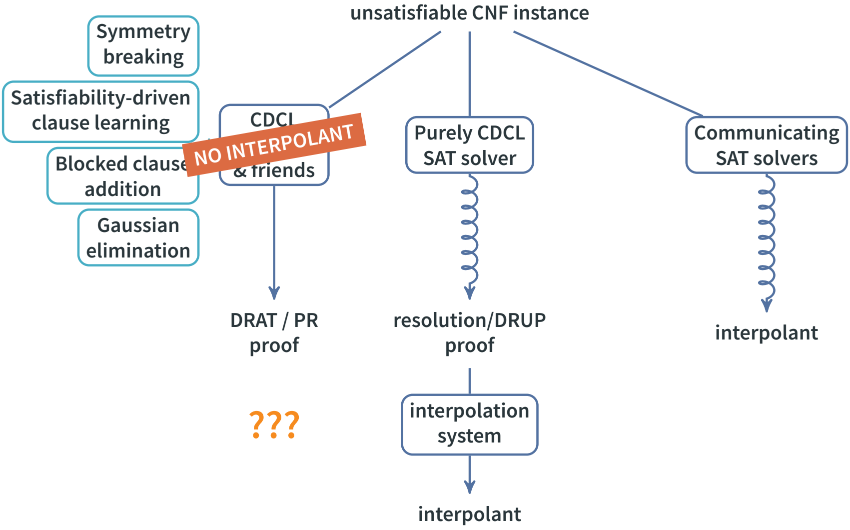

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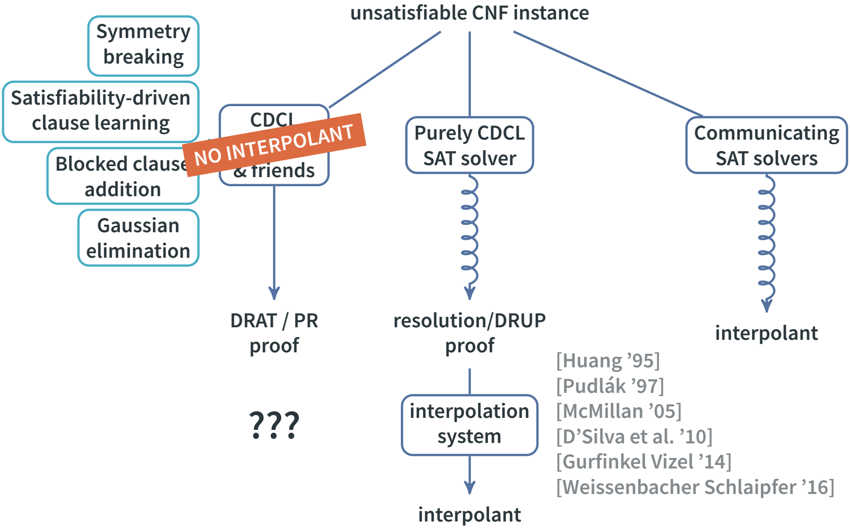

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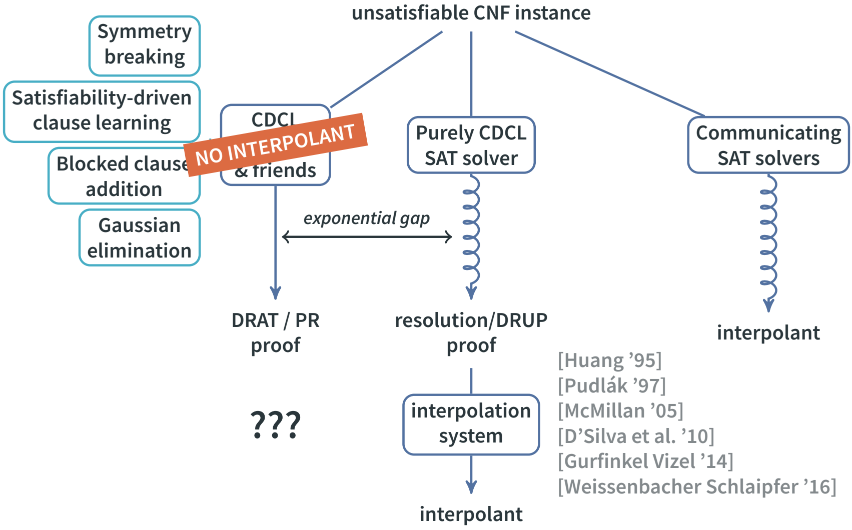

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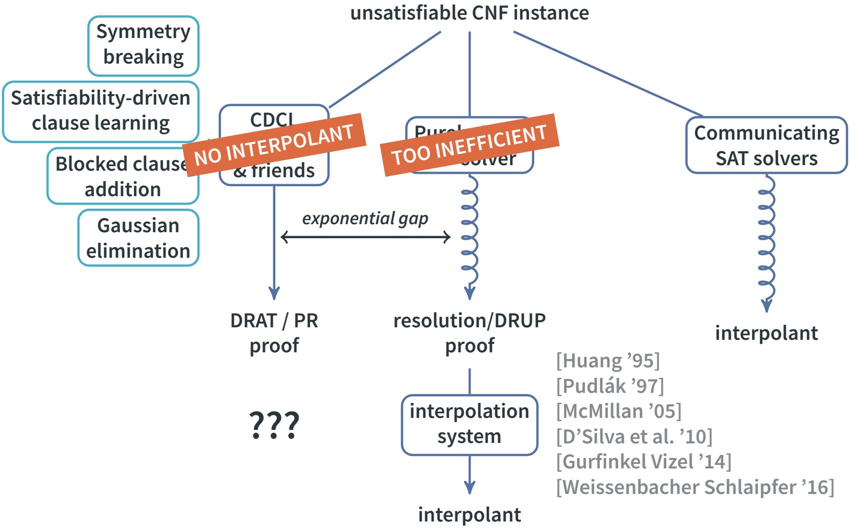

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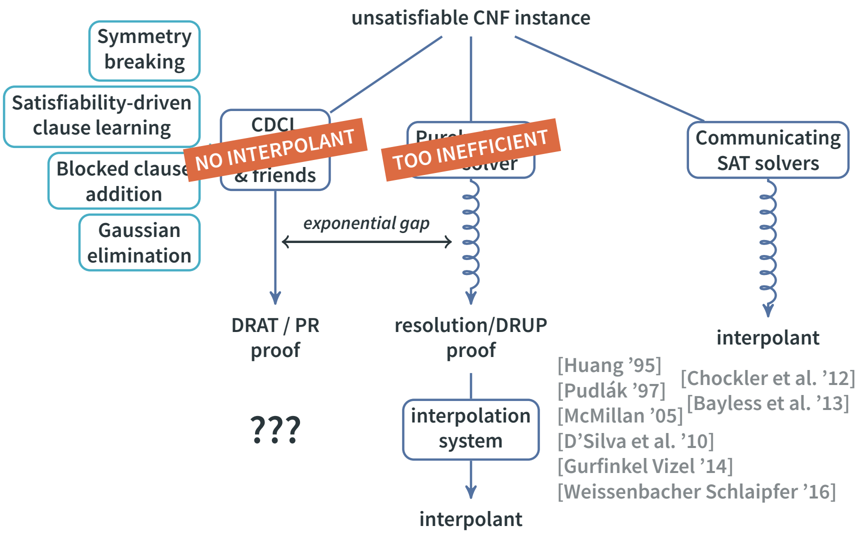

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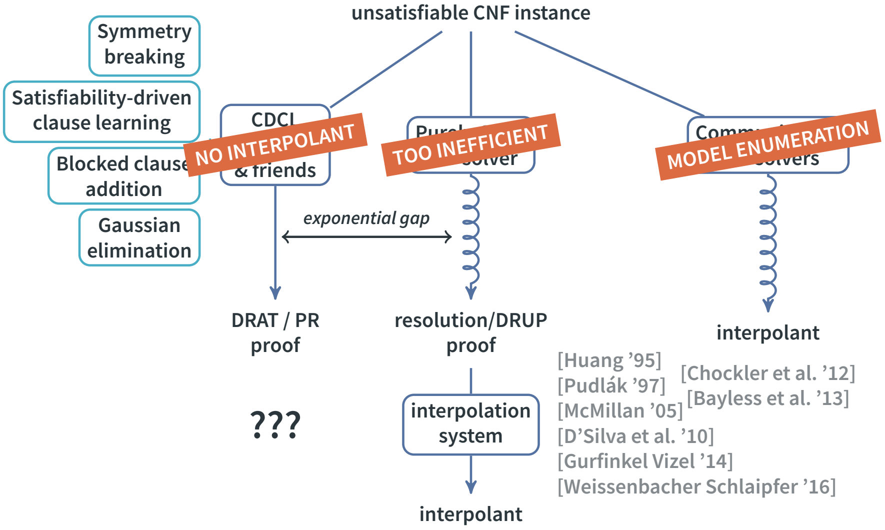

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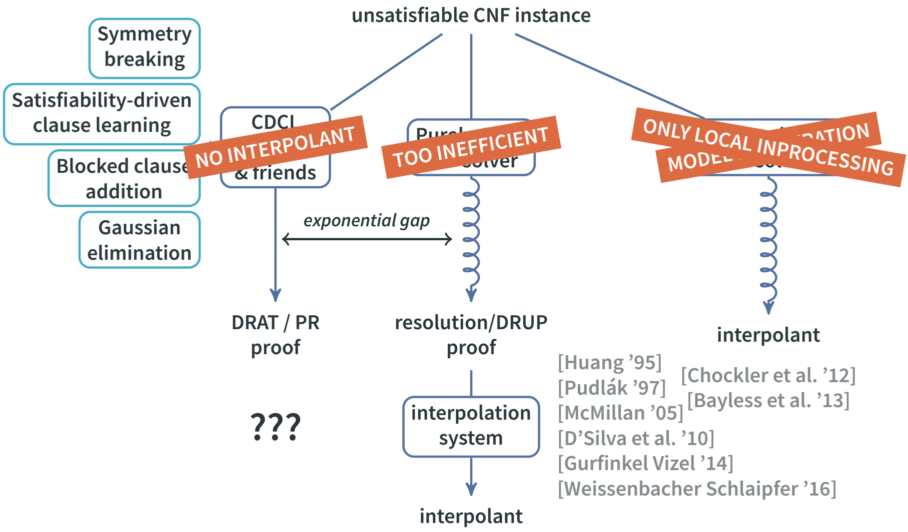

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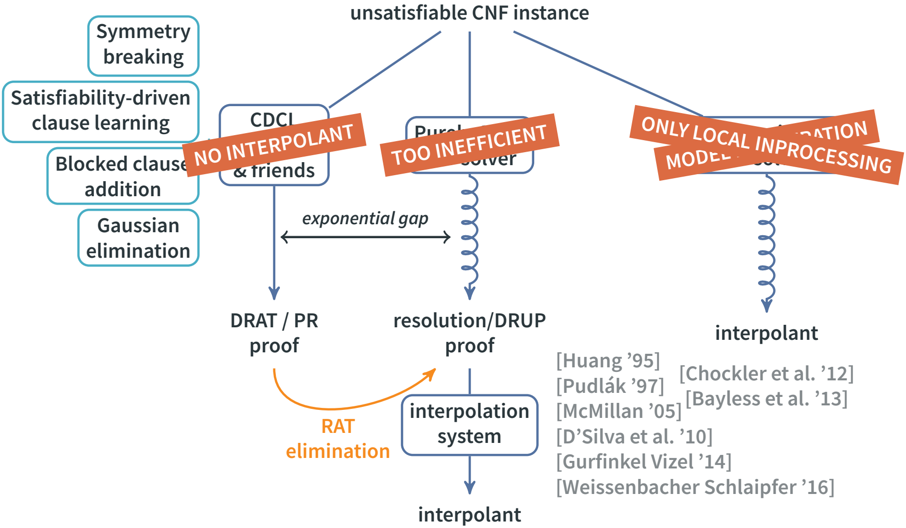
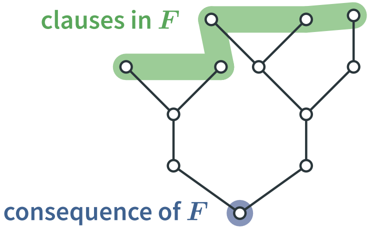


Proof systems for SAT solvers

DRUP proofs

Reverse Unit Propagation (RUP)



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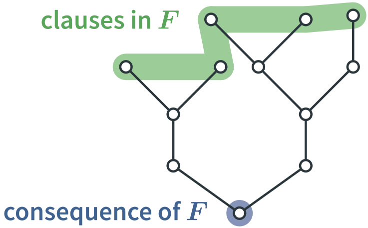

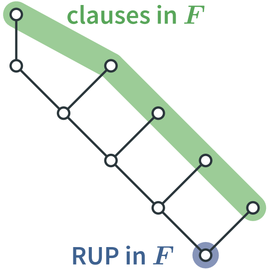

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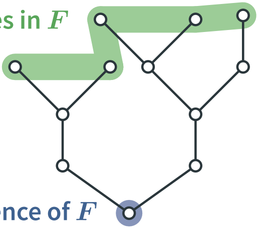

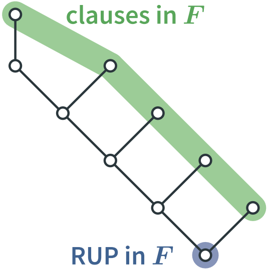

DRUP proof system RUP introduction + arbitrary clause deletion

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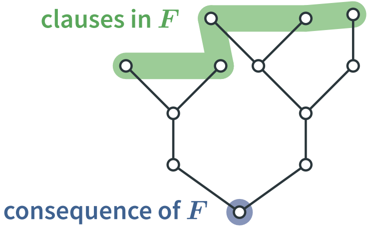

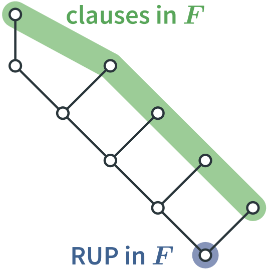

DRUP proof system RUP introduction + arbitrary clause deletion

- Essentially as powerful as resolution [Beame et al. ’04]
- Interpolants can be easily generated [Gurfinkel Vizel ’14]

Resolution asymmetric tautologies

A clause C is a resolution asymmetric tautology (RAT) in a CNF formula F upon a literal l if every resolvent $C \otimes D$ upon l, where $D \in F$, is a RUP in F

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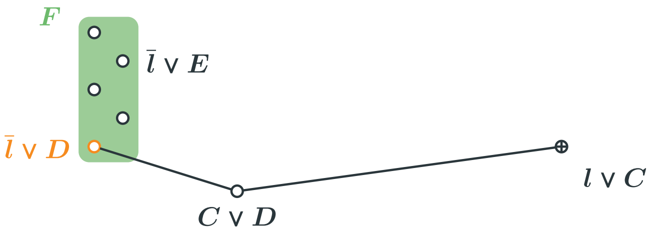

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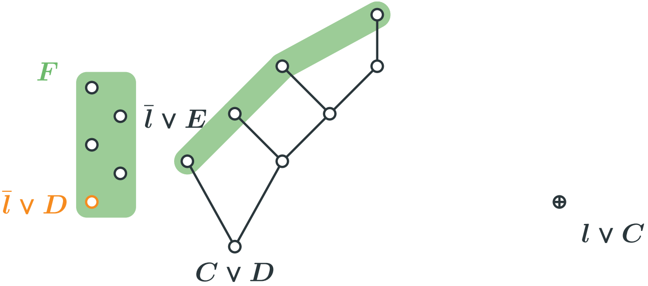

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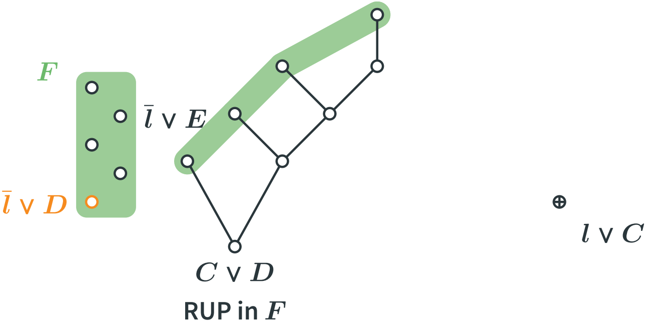

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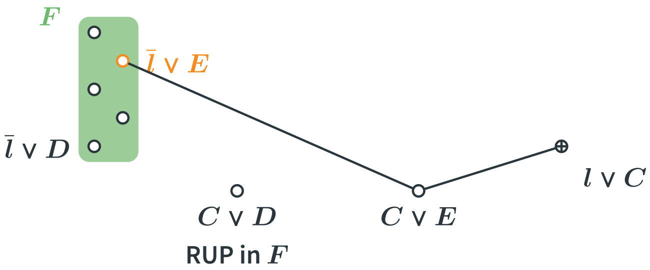

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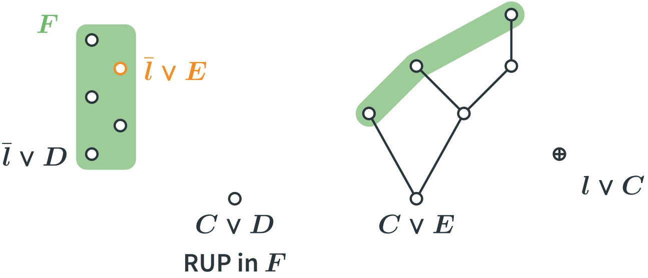

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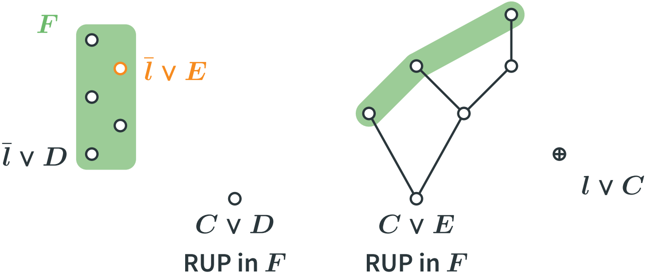

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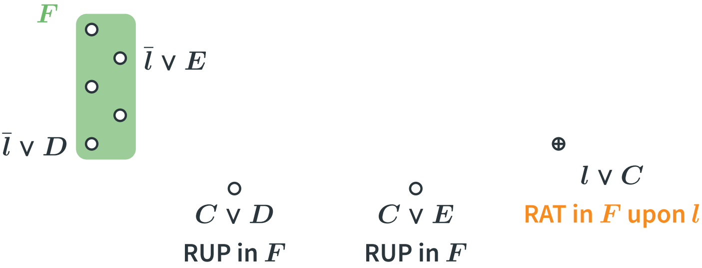

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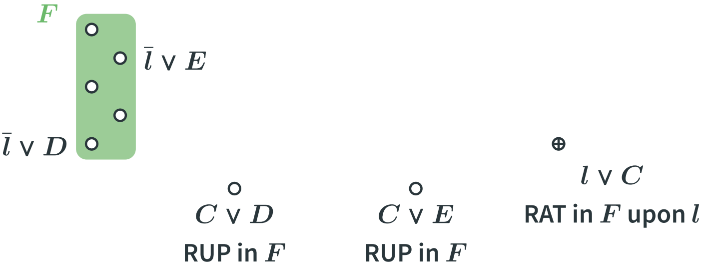

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RAT introduction can be used as an inference rule of a proof system

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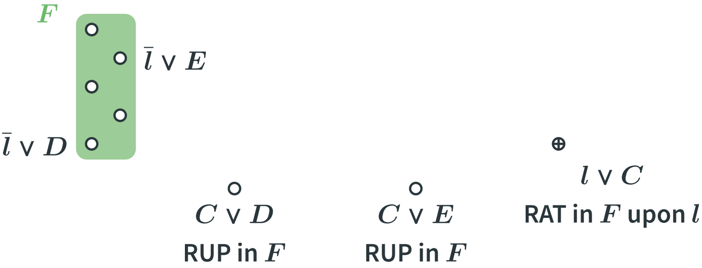

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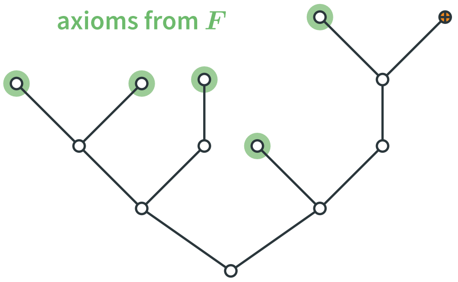
DRAT proof system

RUP introduction + RAT introduction + arbitrary clause deletion

- polynomially simulates extended resolution
  [Heule Kiesl Rebola-Pardo ’18]

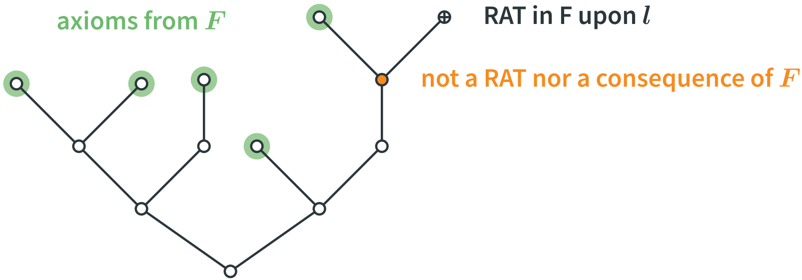
Interpolation from DRAT proofs

RATs, consequences and latency



Why does RAT work? Eventually, some successor of every RAT becomes a consequence

RATs, consequences and latency



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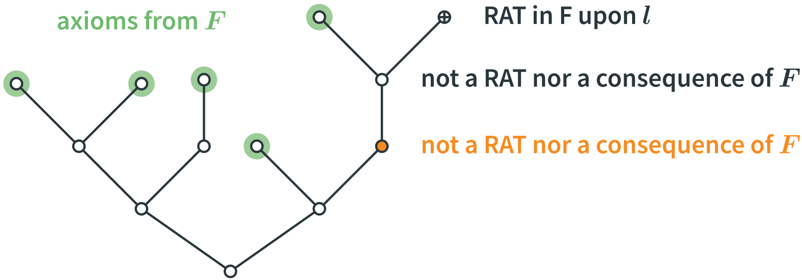

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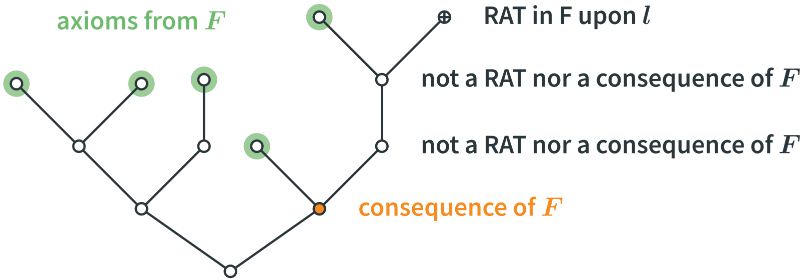

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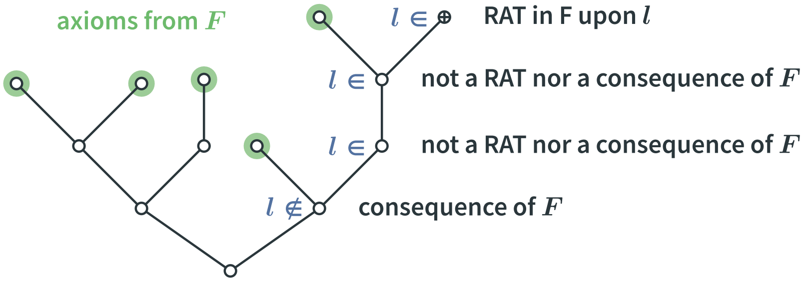

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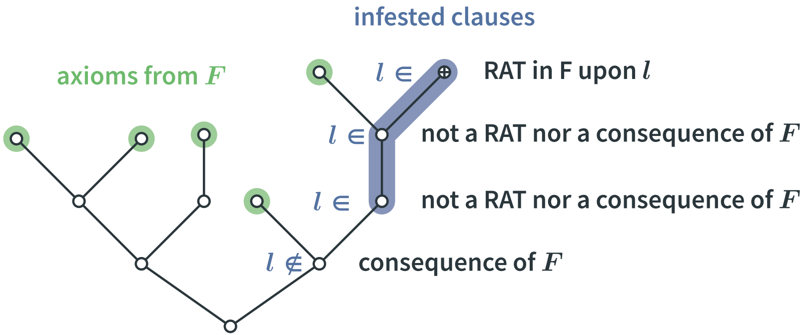

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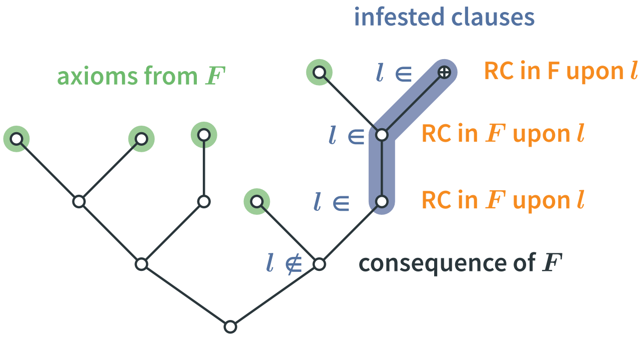

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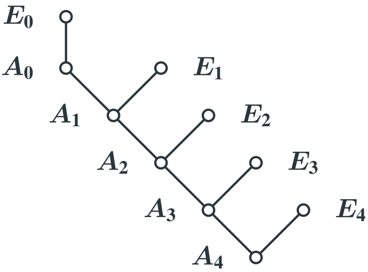
Question Can we obtain a resolution proof of that consequence clause?

Infested clause elimination



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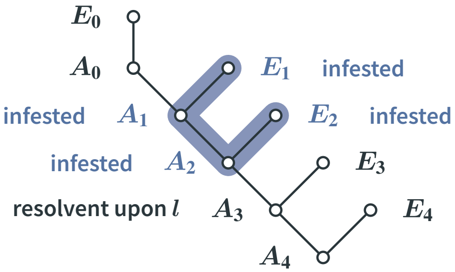

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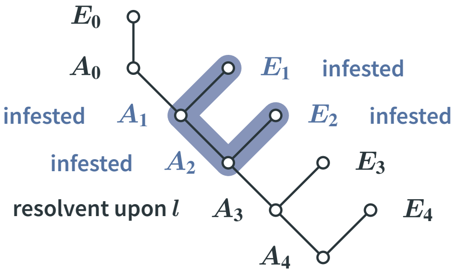

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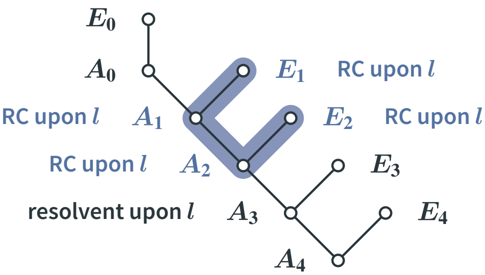

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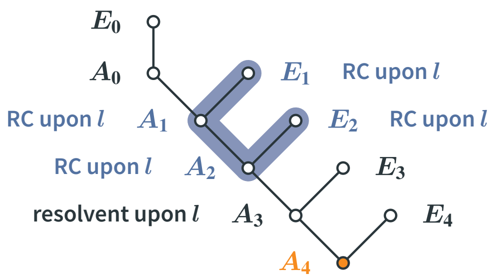

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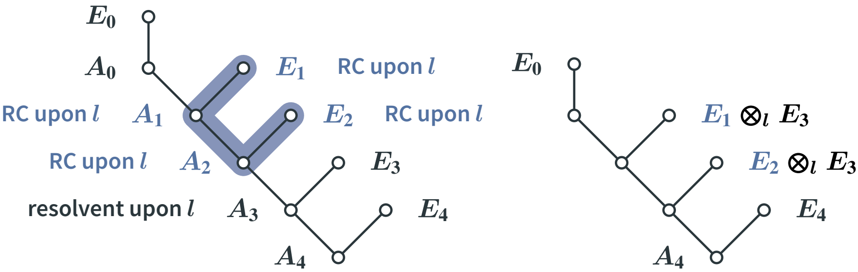

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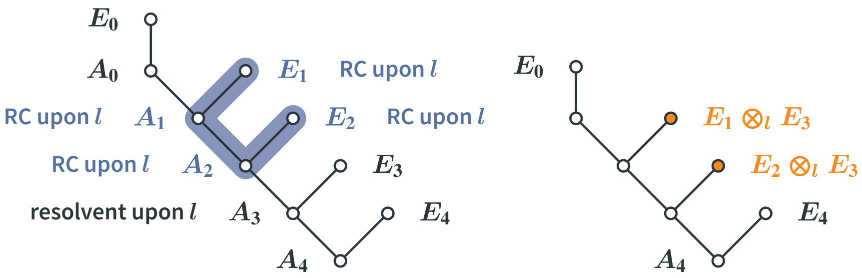

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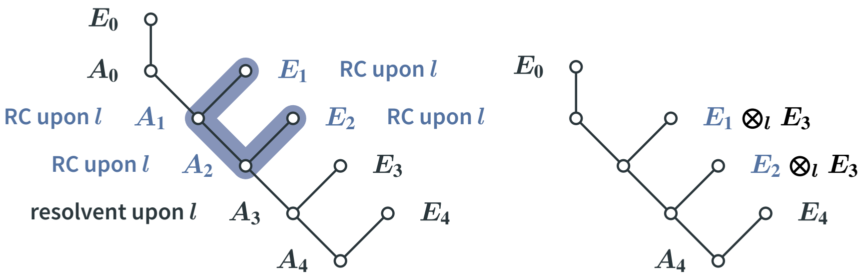

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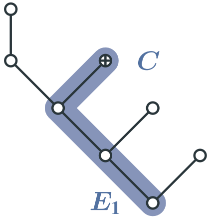

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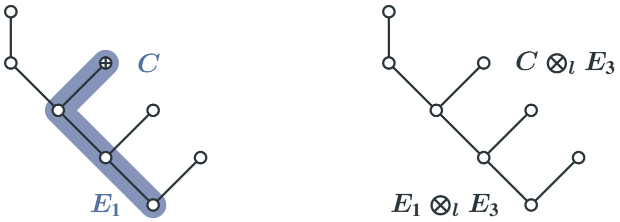


Resolution consequence every resolvent upon l is a consequence of F

all infested clauses are RCs upon l

Goal derive A_4 without using infested clauses

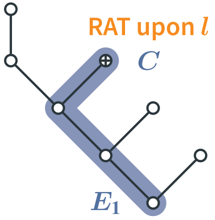
$\Rightarrow$ derive $E_1 \otimes_l E_3$ and $E_2 \otimes_l E_3$ without using infested clauses

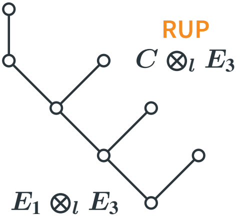
Elimination repeat until only clauses $C \otimes_l D$ need to be derived

C is the original RAT upon l in F

D is a clause in F

Infested clause elimination





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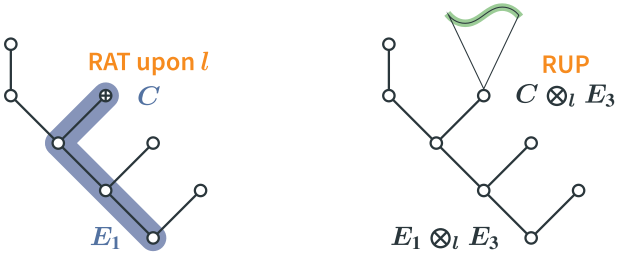

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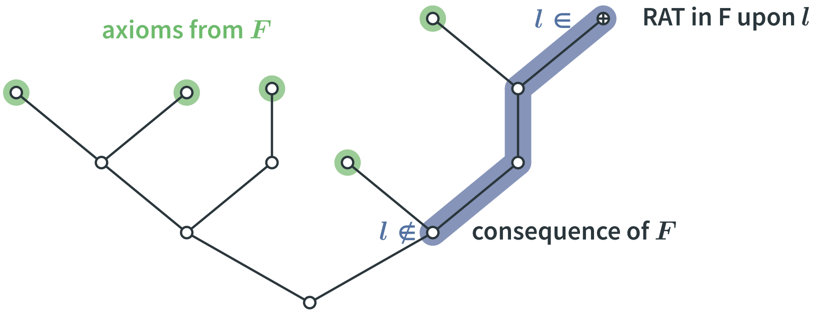
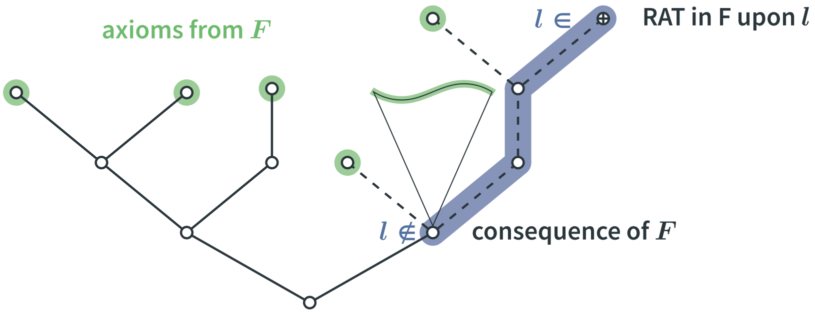
Conclusion

Interpolation through RAT elimination



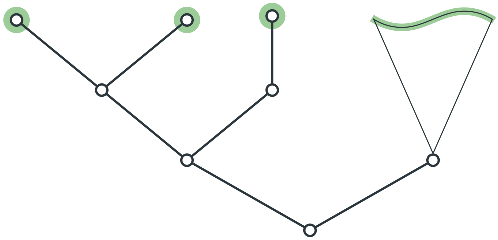
Interpolation through RAT elimination



Interpolant generation from DRAT proofs

Interpolation through RAT elimination

axioms from F



Interpolant generation from DRAT proofs

Interpolation through RAT elimination

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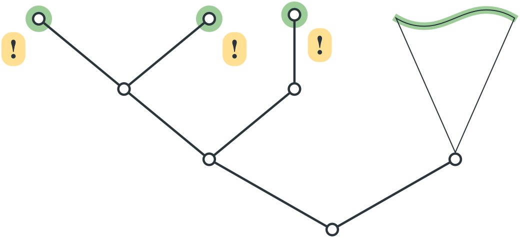
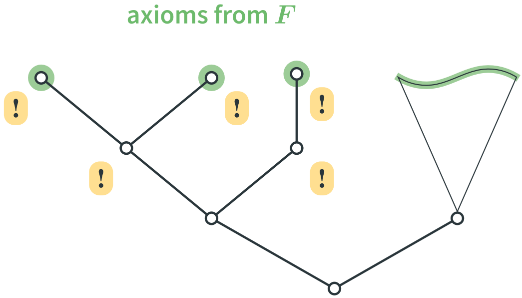
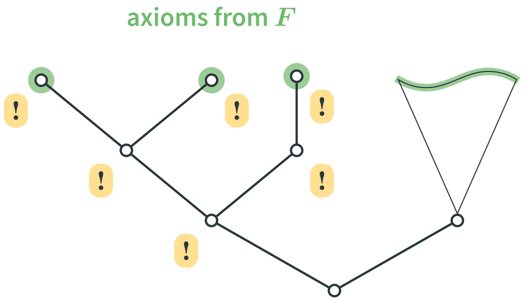


Interpolant generation from DRAT proofs

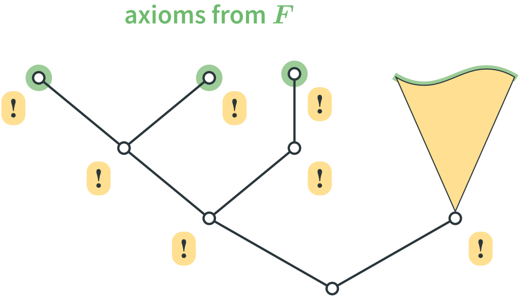
Interpolation through RAT elimination



Interpolation through RAT elimination



Interpolation through RAT elimination



Interpolant generation from DRAT proofs

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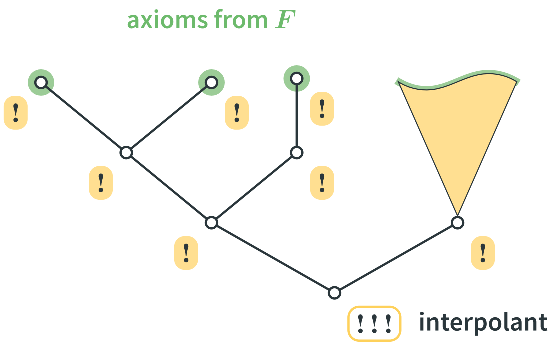
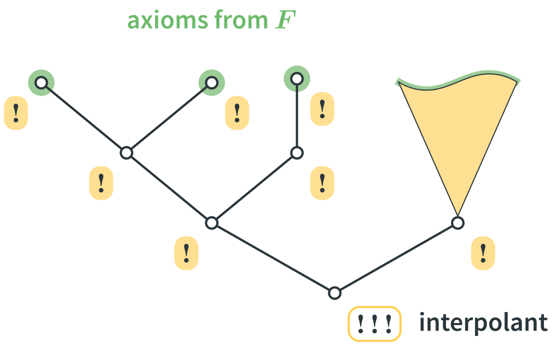


Interpolant generation from DRAT proofs

Interpolation through RAT elimination



Issues

- The interpolant may be exponential with respect to the DRAT proof
  ... but DRAT proofs can be exponentially shorter than DRUP proofs
- Currently we only eliminate RATs one by one
  Open question: can PR clauses be exploited to overcome this?
- Prototype by Martin Matak; Implementation by Adrián Rebola-Pardo (evaluation pending)